GENERATION OF WIDEBAND ELECTROMAGNETIC RESPONSE THROUGH A LAGUERRE EXPANSION USING EARLY TIME AND LOW FREQUENCY DATA

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Abstract: The objective of this paper is to generate a ultrawideband and long temporal response for three-dimensional structures. This is accomplished through the use of a hybrid method that involves generation of early time and low frequency information for the electromagnetic structure of interest utilizing computational available conventional electromagnetic codes. These two, early time and low frequency information are mutually complementary and contain all the necessary information for an ultrawideband response for a sufficient record length. The time domain response is modeled as a Laguerre series expansion. The frequency domain response is also expressed in an analytic form using the same expansion coefficients used in modeling of the time domain response. The data in both the domains is used to solve for the polynomials coefficients in a data fitting procedure. Once the polynomial coefficients are known, the available data is simultaneously extrapolated in both domains. This approach is attractive because expansions with a few terms give good extrapolation in both time and frequency domains. The computation involved to generate a ultrawideband response is minimal with this method.

1. Introduction: The time and frequency domain responses from three-dimensional objects are considered in this paper. It is assumed that the electromagnetic structures are excited by band-limited functions, such that both the time and frequency domain responses are of finite support for all practical purposes. From a strictly mathematical point of view, a causal time domain response cannot be strictly band-limited and vice-versa. However, a response strictly limited in time can be assumed to be approximately band-limited if the amplitude of the frequency response is too small (outside the region of interest) to be of any consequence.

In computational electromagnetics, one needs to obtain the electromagnetic "fingerprint" of an object. This is equivalent to obtaining the entire impulse response in the time domain or obtaining the transfer function over the entire frequency band. Both of this information requires tremendous computational resources. Here we propose a hybrid approach, which will minimize the computational efforts. The goal is attained by generating early time data using a time domain code and the low frequency information using a frequency domain code, which are not computationally demanding. Then a Laguerre series is fitted to the data in the time and its transform – a polynomial – in the frequency domain. The fit in both time and frequency domains are used to extrapolate the response simultaneously in time and frequency. In this approach, we are not creating any new information but using the existing information to extrapolate the responses simultaneously in time and frequency domain.

It is better to use the Laguerre polynomials instead of the associate Hermite functions [1] even though the later are the eigenfunctions of the Fourier transform operator. The problem with the Hermite expansion is that these polynomials are two sided (- , +) and hence the choice of the origin of the expansion of the causal time domain functions by a Hermite series is very critical. In contrast, the Laguerre series is defined only over the interval [0, +] and hence are considered to be more suited for the problem at hand, as they naturally enforce causality.

2. Formulation: Consider the set of functions [2],

$$L_n(t) = \frac{1}{n!} e^x \frac{d^n(t^n e^{-t})}{dt^n} \qquad n \geq 0 ; t \geq 0 \tag{1}$$

These are the Laguerre functions of order n. They are causal [3], i.e., exist for t 0. They can also be computed in a stable fashion recursively through

$$L_0(t) = 1;\ L_1(t) = 1 - t;\ L_n(t) = \frac{2n-1-t}{n} L_{n-1}(t) - \frac{n-1}{n} L_{n-2}(t) \qquad n \geq 2, \quad t \geq 0 \tag{2}$$

The Laguerre functions are orthogonal as

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$$\int_0^{\infty} e^{-1} L_n(t)\, L_m(t)\, dt = \delta_m = \begin{cases} 1 & m = n \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

An orthonormal basis function set can be derived from the Laguerre functions through the representation

$$\phi_n(t,\ell) = e^{-t/2}\, L_n(t/\ell) \tag{4}$$

where is a scaling factor. A causal electromagnetic response function x(t) at a particular location in space for t 0 can be expanded into a Laguerre series as

$$x(t) = \sum_{n=0}^{\infty} a_n\, \phi_n(t,\ell_1) \tag{5}$$

These functions can approximate causal responses quite well and by varying the scaling factor , the support provided by the expansion can be increased or decreased. As can be seen, the Laguerre functions are causal and also their modality (number of local maximas and minimas) increases with the increase in order.

A signal with compact time support can be expanded as

$$x(t) = \sum_{n=0}^{N} a_n\, \phi_n(t,\ell_1) \tag{6}$$

The Fourier Transform of the above expression can be evaluated as

$$X(f) = \sum_{n=0}^{N} \frac{a_n \left(-\frac{1}{2} + j\frac{f}{\ell_2}\right)^n}{2\pi\, d_2 \left(\frac{1}{2} + j\frac{f}{\ell_2}\right)^{n+1}} \tag{7}$$

where $\ell_2 = \frac{1}{2\pi d_1}$ and $j = \sqrt{-1}$. The choice of the scaling factor $_1$ is crucial, because it also affects $_2$ and these two decide the amount of support given by the Laguerre functions to the time and frequency domain responses respectively. Given initial time domain data and low frequency data, with a proper choice of N (order of expansion) and $_1$ (scaling factor), it is possible to simultaneously extrapolate the function in both domains. The value of N is decided by the time-bandwith product of the extrapolated waveform and is called the dimensionality of the waveform. The coefficients for the Laguerre expansion are obtained by solving a total least-squares problem, using Singular Value Decomposition (SVD) [4].

Let M_1 and M_2 be the number of time and frequency domain samples that are given for the functions x(t) and X(f), respectively. Here X(f) is considered to be the Fourier transform of x(t).

Then the matrix representation of the time domain data, utilizing equation (6), would be

$$\begin{bmatrix} \phi_0(t_1,\ell_1) & \phi_1(t_1,\ell_1) & \cdots & \phi_{N-1}(t_1,\ell_1) \\ \phi_0(t_2,\ell_1) & \phi_1(t_2,\ell_1) & \cdots & \phi_{N-1}(t_2,\ell_1) \\ \vdots & \vdots & \vdots & \vdots \\ \phi_0(t_{M_1},\ell_1) & \phi_1(t_{M_1},\ell_1) & \cdots & \phi_{N-1}(t_{M_1},\ell_1) \end{bmatrix}_{M_1 \times N} \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_{N-1} \end{bmatrix}_{N \times 1} = \begin{bmatrix} x(t_1) \\ x(t_2) \\ \vdots \\ x(t_{M_1}) \end{bmatrix}_{M_1 \times 1} \tag{8}$$

Similarly in the frequency domain one obtains equation (9) as shown in the next page.

By combining, the two equations, given by (8) and (9), we solve for the N unknown coefficients of the expansion a_i for the given set of data points in time and frequency.

$$
\begin{bmatrix}
\dfrac{1}{\frac{1}{2}+j\frac{f_1}{\ell_2}} & \dfrac{\left(-\frac{1}{2}+j\frac{f_1}{\ell_2}\right)}{\left(\frac{1}{2}+j\frac{f_1}{\ell_2}\right)^2} & \cdots & \dfrac{\left(-\frac{1}{2}+j\frac{f_1}{\ell_2}\right)^{N-1}}{\left(\frac{1}{2}+j\frac{f_1}{\ell_2}\right)^N} \\
\dfrac{1}{\frac{1}{2}+j\frac{f_2}{\ell_2}} & \dfrac{\left(-\frac{1}{2}+j\frac{f_2}{\ell_2}\right)}{\left(\frac{1}{2}+j\frac{f_2}{\ell_2}\right)^2} & \cdots & \dfrac{\left(-\frac{1}{2}+j\frac{f_2}{\ell_2}\right)^{N-1}}{\left(\frac{1}{2}+j\frac{f_2}{\ell_2}\right)^N} \\
\vdots & \vdots & \vdots & \vdots \\
\dfrac{1}{\frac{1}{2}+j\frac{f_{M_2}}{\ell_2}} & \dfrac{\left(-\frac{1}{2}+j\frac{f_{M_2}}{\ell_2}\right)}{\left(\frac{1}{2}+j\frac{f_{M_2}}{\ell_2}\right)^2} & \cdots & \dfrac{\left(-\frac{1}{2}+j\frac{f_{M_2}}{\ell_2}\right)^{N-1}}{\left(\frac{1}{2}+j\frac{f_{M_2}}{\ell_2}\right)^N}
\end{bmatrix}_{M_2\times N}
\begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_{N-1} \end{bmatrix}_{N\times 1}
= 2\pi d_2 \begin{bmatrix} X(f_1) \\ X(f_2) \\ \vdots \\ X(f_{M_2}) \end{bmatrix}_{M_2\times 1} \tag{9}
$$

3. Numerical Example: A program to evaluate the currents on an arbitrarily shaped closed or open body using the Electric Field Integral Equation (EFIE) in the frequency domain is used [5] to genarte the low frequency response. We also use another code, a Time Domain Electric Field Integral Equation [6] to generate the early time response. We utilize the same surface-patching scheme for both domains, hence eliminating some of the effects of discretization from this study. The triangular patching approximates the surface of a scatterer with a set of adjacent triangles. The current perpendicular to each non-boundary edge is an unknown to be solved for.

The Gaussian pulse illuminating the structure is assumed to be of the form

$$
\bar{E}^{inc} = \bar{u}_i \frac{1}{\sigma\sqrt{\pi}} E_0 e^{-r^2} \quad \text{with} \quad \gamma = \frac{(t - t_0 - \bar{r}\cdot\bar{k})}{\sigma} \tag{10}
$$

$\bar{u}_i$ is the unit vector that defines the polarization of the incoming plane wave.

E_0 is the amplitude of the incoming wave.

t_0 is a delay and is used so that the pulse rises smoothly from 0 for time $t < 0$ to its value at time t.

$\bar{r}$ is the position of an arbitrary point in space.

$\bar{k}$ is the unit wave vector defining the direction of arrival of the incident pulse.

ó^2 is the spread factor of Gaussian input pulse.

To find the frequency response of any structure to the above Gaussian plane wave, the frequency response of the system is multiplied by the spectrum of the Gaussian plane wave. The spectrum is given by

$$
F(j\omega) = \frac{E_0}{c}\, e^{-\left[\frac{(\omega\sigma)^2}{4c^2} + j\omega t_0\right]} \qquad \omega = 2\pi f \tag{11}
$$

In all our computations, E_0 is chosen to be 377 V/m. The time step (Ät) is dictated by the discretization used in modeling the geometry of each example. The frequency step (Äf) is 2MHz.

In all the examples, the extrapolated time domain response is compared to the output of the time domain response obtained from the Marching-on-in-Time (MOT) program [6] and the extrapolated frequency domain response is compared to that of the Method of Moments (MOM) program [5]. In all the plots, dashed line refers to the extrapolated response using Laguerre expansions; while the solid line refers to the data obtained from the MOT or MOM program.

A plate-sphere combination is considered, with the sphere of radius 1m centered at the origin and separated by 5 m. The actual discretization is shown in Figure 1. The excitation arrives from $\theta = \frac{\pi}{2}$, $\phi = 0°$ i.e., along the negative x-direction . $\bar{u}_i$ is along the x-axis. In this example, $\sigma = 2.359$ ns and $t_0 = 9.20$ ns. The time step used in the MOT program is 0.484 ns. The edge under consideration is on the plate, along the y direction and close to center to the center. The time domain data is obtained using the MOT algorithm from t = 0 to t = 145ns (300 data points). In addition, the frequency domain data is obtained using the MOM program from DC to f = 298 MHz (150 data points). Using the first 80 time data points (upto 38.67ns) and the first 50 frequency data points (upto 98 MHz), the time domain response is extrapolated to 300 points and the frequency domain response is extrapolated to 150 points. The order of the expansion is chosen to be 50. From Figure 2, it can be seen that the time domain reconstruction is agreeable to the actual MOT data. From Figure 3, the real and imaginary parts of the frequency domain response also have reasonable good reconstruction using the Laguerre expansions.

4. Conclusion: This paper deals with the problem of simultaneous extrapolation in time and frequency domain using only early time and low frequency data so as to generate either an ultrawideband response in the frequency domain or a long transient response. The generation of wideband response has been accomplished through the use of Laguerre expansion, which are inherently causal and thus fit the time domain data better than the associate Hermite functions. The computation involved is minimal because we require only early time and low frequency information. In addition, we need to solve a small matrix equation. This, coupled with the fact that expansions of order around 50 give good representation of the signals in both domains, ensures that this method is computationally very efficient.

5. References

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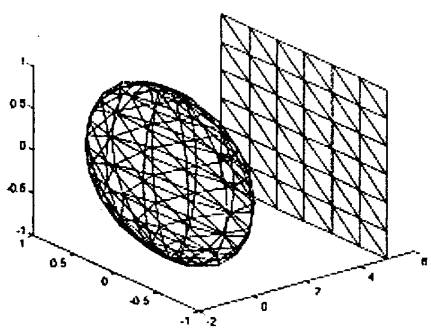

Fig. 1: Discretization of the plate-sphere structure.

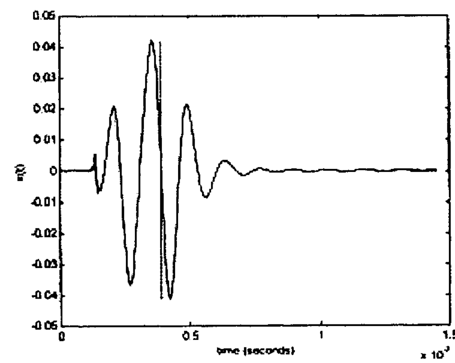

Fig. 2: Time domain response at one of the edges on the plate

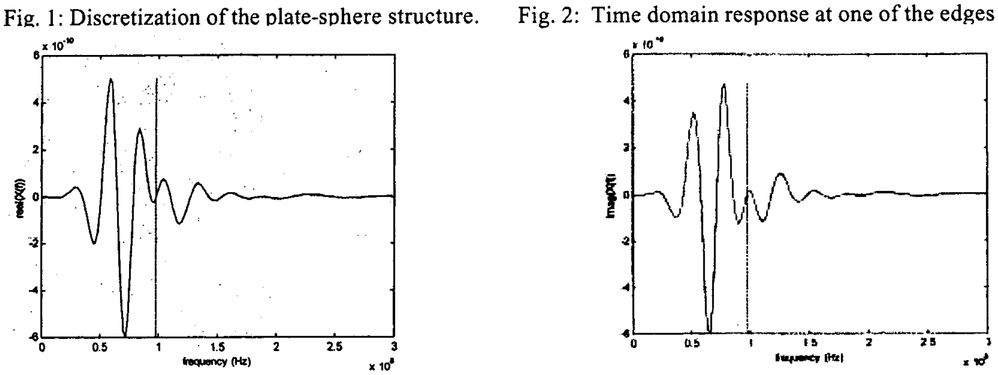

Figure 3: Frequency response at one of the edges on the plate – real & imag parts.